Triangulating How Things Look

JOHN MORRISON

Abstract: Suppose you're unable to discriminate the colors of two objects. According to the triangulation view, their colors might nonetheless look different to you, and that's something you can discover as a result of further comparisons. The primary motivation for this view is its apparent ability to solve a puzzle involving a series of pairwise indiscriminable objects. I argue that, due to visual noise, the triangulation view doesn't really solve the puzzle.

1. Introduction

Suppose you want to know whether two objects look exactly the same color. You place them against a solid white background and are unable to tell their colors apart. Should you automatically infer that they look exactly the same color? The *triangulation view* is that you shouldn't. You should first search for a third object that's discriminable from only one of the original objects in respect of color. If you find such an object, infer that the original objects do *not* look exactly the same color. If you're unable to find such an object, and your search has been sufficiently thorough, infer that they look exactly the same color. Either way, you're supposed to know whether the objects look exactly the same color, at least in most cases.

A number of well-known philosophers have objected to the triangulation view, including Dummett (1975), Wright (1975), Varzi (1995), and Fara (2001). But their objections either rely on questionable assumptions or don't really identify what's wrong with the view. Because it's important to reject the wrong view for the right reasons, I will suggest another reason to reject the triangulation view. My objection will focus on random variations in our visual system. This phenomenon ('visual noise') is well known and frequently discussed by vision scientists,[1] but rarely acknowledged by philosophers.[2] A subsidiary goal of this article is to convince philosophers to pay more attention to visual noise and its implications.

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Address for correspondence: 326b Milbank Hall, 3009 Broadway, New York, NY 10027, USA.
Email: jmorrison@barnard.edu

[1] Green and Swets, 1966, Chapter 6, is the standard reference. See also McNicol, 1972, Chapter 1.

[2] Hardin (1988, pp. 215f) and Hellie (2005, pp. 492–495) are exceptions. Chuard (2010, pp. 186–187) discusses several consequences of visual noise, like Weber's Law, without identifying their underlying cause (see Treisman, 1964). None of these authors discuss the triangulation view.

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In the next section I'll explain why the triangulation view is worth taking seriously. In particular, I'll introduce a vexing puzzle that it's designed to solve. I'll then argue that due to visual noise it doesn't really solve the puzzle, thereby undermining its primary motivation.

2. The Puzzle

To make the puzzle more concrete, I'm going to tell a story. My story invokes the notion of pairwise indiscriminability (hereafter just: indiscriminability). I'll say more about this notion later. For now, let's just rely on our pre-theoretic understanding (roughly: 'can't tell them apart').

Poor Jonah! He wanted to select exactly the right shade of paint for his new kitchen cabinets. Unfortunately, at the paint store, he discovered that he was not up to the task. Here's what happened: a few minutes after entering the store, he was drawn to a bright red paint chip. He put it in his pocket and kept looking. A few minutes later, he was drawn to another. And a few minutes later, yet another. He then pulled out the first two chips, placed them against a solid white background, and judged:

(1a) The first and second chips have indiscriminable colors in this context.

On this basis he inferred:

(1b) The first and second chips look exactly the same color in this context.

After returning those chips to his pocket he then pulled out the second and third chips, placed them against the same white background, and judged:

(2a) The second and third chips have indiscriminable colors in this context.

On this basis he inferred:

(2b) The second and third chips look exactly the same color in this context.

Concluding that all three chips must be coated with the same paint, he prepared to make his purchase and leave. But he then compared the first and third chips against the same white background and, to his great surprise, judged:

(3a) The first and third chips have discriminable colors in this context.

On this basis he inferred:

(3b) The first and third chips do not look exactly the same color in this context.

Just as two objects viewed simultaneously can look exactly the same color, so can one object when viewed at different times. Jonah viewed the first chip twice, once when it was next to the second chip and once when it was next to the third chip. These two contexts were extremely similar: same lighting, same background, same viewing distance, and so on. Knowing this, Jonah inferred that the first chip looked exactly the same color both times he viewed it—its coloring looked constant. He likewise inferred that the color of the second and third chips looked constant. It will be helpful to expresss these inferences in a slightly different way. Let's say that a group of contexts is 'stable' for an object if the object looks exactly the same color when it's viewed in any of those contexts. We can express Jonah's inference:

(4b) These three contexts were stable for these three chips.

He then recalled that as a schoolboy he was taught:

TRANSITIVITY
For all objects in stable contexts: if x and y look exactly the same color and y and z look exactly the same color, then x and z look exactly the same color.

But this principle is mutually inconsistent with (1b)–(4b). TRANSITIVITY, (1b), (2b), and (4b) together imply that the first and third chips look exactly the same color. But (3b) says that the first and third chips do *not* look exactly the same color. Confused by the inconsistency, Jonah went home empty-handed.

Jonah's predicament is familiar. It is also easy to replicate in a laboratory. Our job is to figure out which of the inconsistent claims is false and, if possible, explain how Jonah can know it is false.

The triangulation view offers an especially attractive response to the puzzle. The triangulationist insists that Jonah is in a position to know that (1b) and (2b) are false. She claims that (2a) and (3a):

(2a) The second and third chips have indiscriminable colors;
(3a) The first and third chips have discriminable colors

should have prompted Jonah to infer:

(1c) The first and second chips do not look exactly the same color

and that (1a) and (3a) should have prompted Jonah to infer:

(2c) The second and third chips do not look exactly the same color

and that, as before, (3a) alone should have prompted Jonah to infer:

(3c) The first and third chips do not look exactly the same color.

(1c)–(3c) are jointly consistent with TRANSITIVITY. Problem solved?

I'll argue that in many cases triangulating doesn't actually give us a way of knowing how things look. As noted above, my argument will center on the rarely mentioned fact that, like all measuring systems, our visual system is noisy.

There are a number of other responses. But the obvious alternatives all have drawbacks (see Morrison, 2015), and, as we'll see, the problem with the triangulation response is subtle, so it's no surprise that this response has received so much attention.

Before proceeding, I want to point out that the triangulation response is about our *knowledge* that two objects look exactly the same color. It is officially noncommittal about what *makes it the case* that two objects look exactly the same color. Of course, the most natural thing for a triangulationist to say is: Two objects look exactly the same color if and only if there is no third object discriminable from only one of them. But a triangulationist is free to say something else, and, in any case, my arguments won't depend on any assumptions about this metaphysical issue.

I also want to distinguish our puzzle from a similar puzzle. The other puzzle is about color qualia, which are supposed to be the qualitative properties of our experiences when we look at colored objects.[3] Some philosophers claim there is a series of experiences such that we can't discriminate the qualia of successive experiences but we can discriminate the qualia of the first and last experience in the series. Some respond by adopting views that are like the triangulation view. Most famously, Goodman (1966) claims we can always triangulate qualia. However, one difference between this view and the triangulation view is that while visual noise unquestionably interferes with our access to the colors of external objects like paint chips, it might not interfere with our access to qualia. Therefore, the problem I'll develop for the triangulation view might not generalize to Goodman's view.

3. Terminology

The phrase 'look exactly the same color in this context' is central to this puzzle and the triangulation view. It is therefore worth taking a minute to clarify what it means.

To start, the same object can look different to you and me, even in the same context. Thus, claims about how things looks are always relative to an observer. With respect to our puzzle, the claims are relative to Jonah.

Next, many philosophers claim there are three or more different senses of 'looks'.[4] We don't need to engage with that discussion. All that's important is that 'looks' picks out a relation that satisfies the following four conditions: First, it relates us to paint chips and other external objects as opposed to inner, mental objects like experiences or sense-data. Second, it is representational in some sense—it corresponds to how things *seem* to us. For this reason, as another way of saying that the chip looks

[3] There are other uses of 'qualia'. See Tye, 2013.

[4] See Breckenridge, 2007, pp. 11–19, for an overview and critical discussion.

bright red to Jonah, I will sometimes say that Jonah visually represents the chip as bright red.[5] Third, it corresponds to a conscious mental state rather than unconscious information processing in that, for every change in how something looks, there is a corresponding change in our conscious mental state. Fourth, it is independent of our beliefs in that, for example, a wall can look gray even if we believe it is white.

We can accommodate different senses of 'looks' as long as they satisfy these conditions (with a possible exception that I'll discuss later). Suppose you're looking at a white wall that's partially covered by a shadow. Arguably, there's a sense in which the covered part of the wall looks darker, and there's a sense in which the entire wall looks to be the same shade of white. Rather than choose between these two senses of 'looks', we can leave the discussion sufficiently general that it applies to both of them. If it turns out that there's only one sense of 'looks', all our uses of 'looks' should be interpreted as having that sense.

Let's now turn to the compound phrase 'look exactly the same color'. I'm using this phrase with a familiar and implicit restriction. We might think of the colors as forming a pyramid. Near the top of the pyramid are colors like red, yellow, and blue. Below them are colors like magenta, rose, canary, lemon, aqua, and navy. At the bottom are the most fine-grained/exact/determinate colors (hereafter just: fine-grained colors). There are some theories of color according to which the colors at the bottom are so fine-grained that no human being could ever individually represent them (e.g. Byrne and Hilbert, 2003). Nonetheless, when we say that two things look exactly the same color, we're usually talking about the lowest level at which colors are represented, and that's how I'm using the expression. That is, I'm using 'look exactly the same color' so that it is restricted to the most fine-grained colors that we can represent.

What is the lowest level at which colors are represented? Just as there might be no moment at which the letters on an eye chart suddenly become invisible as it moves farther and farther away, so there might be no level in the color pyramid at which colors suddenly become too fine-grained to be represented. My arguments will be compatible with this kind of fuzziness, although I will often talk as if it doesn't exist.

Finally, there is a scope ambiguity in the claim that two chips look exactly the same color to Jonah. According to the wide-scope interpretation, there is a fine-grained color that Jonah represents each chip as instantiating. According to the narrow-scope interpretation, Jonah represents the chips as the same with respect to their fine-grained colors. I have in mind the narrow-scope interpretation.

4. Intuition

As I said, the triangulation response initially seems like an attractive response to the puzzle. One of its most attractive features is that, unlike other responses, it seems

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[5] Some philosophers will object, because they deny that our experiences represent colors (e.g. Travis, 2004). I won't have the opportunity to engage with their concerns in this article.

to preserve one of our core intuitions: that we can know whether two objects look exactly the same color by introspection. I'll later argue that, due to visual noise, the triangulation response doesn't really preserve this intuition. It will therefore be helpful to start by supporting and clarifying the relevant intuition.

In support of this intuition, consider that we often seem to know that two objects look exactly the same color without asking other people or consulting scientific instruments. In contrast, it is far less intuitive that we can know by introspection whether objects *really are* exactly the same color, because, for all we know, our experiences might not be veridical. Nonetheless, at least under ordinary circumstances, we do seem able to tell by introspection that things *look* exactly the same color.

In further support of this intuition, remember that looks-talk is supposed to characterize how things seem to us, and there's something peculiar about cases in which x seems φ to us but we are unable to access this fact. One wonders: In what sense does x still seem φ to us? Perhaps someone can convince us that there's nothing peculiar about such cases. But, all other things being equal, it seems preferable to avoid postulating inaccessible facts about how things seem to us.

Let's now clarify the intuition. To begin, it is part of the relevant intuition that introspection is a *reliable* method for knowing whether two things look exactly the same color. We aren't just making lucky guesses. What's required for reliability, at least in this case? At a minimum:

> We know that two objects look exactly the same color in a context by a *reliable method* only if we're sufficiently likely to believe those objects look exactly the same color in that context using that method.

If someone asks us to compare two objects and we respond that they look exactly the same color, we don't seem to be making an arbitrary guess, as when we arbitrarily guess that a fair coin landed heads. Intuitively, our access to how how things look makes us *likely* to give the correct answer. Thus, it's part of the relevant intuition that we're at least reliable in this sense.

By design, this is a minimal condition. One reason it's minimal is that our knowledge can satisfy it even if there's a non-negligible likelihood that the same method would have produced a false belief. Reliability doesn't entail infallibility. Another reason it's a minimal condition is that reliability is calculated on the basis of how one would respond to those very same objects in the very same context; it is not a function of how one would respond to similar objects, under similar contexts, or in other nearby counterfactuals. It therefore doesn't require one's belief to be sensitive to small changes. It is also worth stressing that I'm not assuming anything about the definition of 'knows'. I'm just reporting a feature of the relevant intuition: that introspection is a reliable method for knowing how things look. Just as our knowledge of basic arithmetic truths might be infallible even if infallibility isn't a necessary condition for knowledge, our knowledge of how things look might be reliable in this sense even if reliability isn't a necessary condition for knowledge.

For all these reasons, this condition should be distinguished from so-called 'safety conditions' on knowledge, like the kind Williamson exploits in his famous argument against the luminosity of most mental states. According to Williamson's safety condition, you know that you feel cold only if you'd still have a correct belief under similar conditions, including conditions in which you feel a tiny bit warmer (see Williamson, 2000). Unlike Williamson's condition, our condition isn't supposed to be constitutive of knowledge, doesn't depend on what we'd believe under similar conditions, and allows us to subsequently respond differently to the same stimulus. Therefore, our condition imposes a much weaker constraint than Williamson's safety condition. This is significant because, like others, I'm persuaded that Williamson's safety condition is implausibly strong (see, e.g., Neta and Rohrbaugh, 2004).[6]

Our intuition should also be distinguished from more standard reliability conditions. Suppose you're searching for metal along the beach using an insensitive detector. In particular, while a piece of metal is unlikely to trigger the detector (hence: it's insensitive), if the detector is triggered, it is likely that there is a piece of metal below. That is, prob(trigger|metal) is low while prob(metal|trigger) is high. According to a standard reliability condition, if the detector is triggered, then your knowledge that there is metal below is nonetheless formed by a reliable method, because the only probability that's relevant is prob(metal|trigger) (see Goldman, 2011). Our condition concerns the other kind of probability. It is like the claim that, if there's metal below, then you're sufficiently likely to know it's there using the detector. This claim is false when working with an insensitive metal detector. But introspecting our own mental states isn't like searching for metal on a beach with an insensitive detector. Intuitively, if two objects look exactly the same color, then that's something we can access through a reliable method.

We just clarified the sense in which introspection is supposed to be a *reliable* method for knowing whether two things look exactly the same color. Let's now clarify two other aspects of the intuition about accessibility. First, to say that by introspection we *can* know whether objects look exactly the same color doesn't imply that we're always ready to exercise that capacity. We might be too tired, too distracted, or too disoriented, for example. On the other hand, it is supposed to be a capacity that most of us actually possess, so it can't involve an extraordinary amount of time. For example, it can't require us to concentrate on our experiences for an

[6] Williamson's anti-luminosity argument implies that we can't know that two objects look exactly the same color. It is therefore inconsistent with the intuition under discussion. Would Williamson's anti-luminosity argument thereby undermine the motivation for the triangulation view, assuming we accepted his safety condition? No. One of the premises in Williamson's argument is that we're unable to discriminate sufficiently small changes, including when we feel a tiny bit warmer (see Williamson, 2000, p. 97), and that presupposes the failure of the triangulation view, because the triangulation view implies we can discriminate such changes (see section five below). Therefore, we need to reject the triangulation view *before* accepting Williamson's argument. If you're a proponent of Williamson's anti-luminosity argument, that's another reason to be interested in objections to the triangulation view.

hour.[7] Where, exactly, is the line between an ordinary and an extraordinary amount of time? It won't matter. The triangulation theorist is free to draw the line wherever she likes. My argument will just presuppose that at some point (a second, a minute, a day, a week, whatever) the triangulation theorist will agree that an extraordinary amount of time has passed.

Second, the relevant intuition does not presuppose that we can know whether objects *look red* by introspection. Imagine a paint chip that is slowly transitioning from red to yellow. At some intermediate point, introspection won't seem like enough to know whether it looks red. In the jargon: whether it looks red will seem 'indeterminate.' Nonetheless, one might still be in a position to know that it looks exactly the same color as another chip.

Now that we've clarified the intuition, let's consider why the triangulation response initially seems to preserve it. Suppose that we know that two objects have indiscriminable colors and we want to know whether they look exactly the same color. According to the triangulation response, that's something we can know by either (i) finding a third object whose color is discriminable from only one of the original objects, or (ii) failing to find such an object after a sufficiently thorough search. A systematic method would be to continuously vary the color of a third object to see if there is a point at which it is discriminable from only one of the original objects. But less systematic methods would be fine too; a painter might consider similarly colored paint chips, a librarian might consider similarly colored books, and a designer might consider similarly colored fabrics. Even if we rarely make comparisons of this sort, because we rarely care whether two objects look *exactly* the same color, it's enough that we could.

In the next section I'll consider a promising but incomplete objection from Dummett (1975). In subsequent sections I will improve on Dummett's objection by filling its explanatory gaps. In particular, I will argue that, due to visual noise, the triangulationist's response does not preserve the intuition about accessibility. Because the triangulation view's primary motivation is its ability to solve the puzzle while preserving such intuitions, I'll conclude that it isn't well-motivated.

5. Dummett's Objection

Dummett's (1975, pp. 322–323) objection is straightforward. As a helpful simplification, let's follow Dummett and shift focus to spatial discrimination. Suppose Eve is visually impaired and can't (pairwise) discriminate locations less than a millimeter apart, but can discriminate locations at greater distances.[8] Consider two locations, l_1 and l_2, that are one nanometer apart. There is a third location, l_3, that's less than a millimeter from l_2 but more than a millimeter from l_1. Therefore, by assumption,

[7] Williamson (1990, pp. 12–13) makes a similar point in response to Hardin, 1988.

[8] Williamson (1994, pp. 238–240) offers a similar example involving a slow-growing tree. Williamson (1990, p. 85) presents the argument schematically.

Eve can discriminate l_3 from l_1 but cannot discriminate l_3 from l_2. The spatial analog of the triangulation view has two implausible consequences. First, it implies that Eve can use l_3 to discover that an edge terminates at l_2 rather than l_1, even though those locations are only one nanometer apart. But it's implausible that someone with such poor spatial acuity can know with such precision where edges terminate, even if she takes into account comparisons with other locations. Second, it implies that Eve can use l_3 to discover that the edge *looks* like it terminates at l_2 rather than l_1. But it's implausible that someone with such poor spatial acuity perceptually represents locations that are so precise. Something must be wrong with the analog of the triangulation view.

The color case is parallel. The primary difference is that colors can differ along more dimensions (light–dark, red–green, and yellow–blue, for example), but that's unimportant, because any two maximally fine-grained colors must differ along at least one dimension. The triangulation view implies that, no matter how bad one's visual acuity, one can nonetheless know that an object has one maximally fine-grained color rather than another. It also implies that one can always know that an object *looks* a particular maximally fine-grained color rather than another. But people with extremely poor visual acuity, including people who can (pairwise) discriminate colors only as different as red and green, can't know anything so precise about the colors of objects, and their experiences don't represent colors that are maximally precise.

This is a powerful objection. It establishes there's something wrong with the triangulation view. But what? The objection doesn't tell us. Like many proofs by contradiction, it isn't explanatory. As a helpful analogy, consider the following argument about time travel:

> If our civilization were to develop the technology to travel backwards in time, then someone would have traveled back in time to tell us about it. Nobody has told us about time travel. Therefore, our civilization will never develop the technology to travel backwards in time.

This argument doesn't explain *why* its conclusion is true. Is it because time travel is impossible? Is there something about our civilization that will prevent us from developing the technology? The argument doesn't say. We're left without any understanding of why the conclusion is true. Dummett's argument is incomplete in the same way. It leaves us without any understanding of why someone can't always rely on triangulation to detect smaller differences than they can pairwise detect. It just tells us that they can't.

I will show why the triangulation view fails due to visual noise.

6. Visual Noise

Because philosophers of perception rarely talk about visual noise, it is worth providing some background.

Visual noise is random variation in our visual system. It has many causes, including chemical processes that alter the amount of light absorbed by the detectors in the back of the eye, neural processes that scramble the signal that the detectors transmit to the brain, and so on. The existence of noise in the visual system is a well-known scientific fact. To quote a vision scientist nearly at random, here's Dennis Pelli: '[N]oise arises in virtually all neural elements of the visual system from photoreceptor to cortex' (1990, p. 12).

How does visual noise manifest itself? To effectively answer this question we'll need to talk about changes in one's experiences. I'm going to describe the relevant changes as changes in qualia. There are many different views of the metaphysics of experience, and this way of talking might not be perfectly ecumenical. But nothing will depend on it. To satisfy representationalists, we could restate the argument in terms of phenomenal colors, which are supposed to be properties that qualia represent (see, e.g., Shoemaker, 1994). To satisfy disjunctivists, we could restate the argument in terms of equivalence classes of indistinguishable experiences. And so on.

Now, close your eyes and closely attend to your visual experience. You'll notice small, constant changes in your color qualia; you'll seem to 'shift' between experiences with slightly different color qualia. This 'shifting' does not go away just because you open your eyes. It just becomes easier to ignore, which is why it often takes practice to notice it.

This is what we should expect if consciousness supervenes on the brain. Suppose that a certain color quale supervenes on a continuous range of neural states, so that you have an experience with that quale whenever your brain is in a neural state in that range. When your brain is in one of the neural states at the beginning or end of that range, your brain will fluctuate between neural states inside and outside the relevant range, resulting in experiences that fluctuate between that quale and other qualia.

Visual noise plays a central role in our puzzle. Consider that just as an object on television might seem motionless even as the pixels flicker, an object might seem to remain exactly the same color even as our qualia fluctuate. In fact, given that visual noise is ubiquitous and that objects often look exactly the same color, that must be the norm. As a result, whether two things look exactly the same color is not a function of our qualia at a particular time. Instead, it is a function of how our qualia fluctuate over time. This has an important consequence that is at the center of the problem for the triangulation response: we can know that two things look exactly the same color only if we can determine that the qualia of the corresponding experiences are fluctuating in the same kind of way. This introduces an uncertainty into our judgments about whether two objects look exactly the same color. The problem for the triangulation response is ultimately due to this uncertainty.

Before I develop the problem, I want to acknowledge a disagreement. Some philosophers claim that *every* change in qualia necessitates a change in what colors things look to have.[9] I doubt that there is any sense of 'looks exactly the same

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[9] See Hellie, 2005, p. 497, and Pelling, 2008, p. 302, for discussion.

color' in which this is true. When you look at a uniformly colored wall under the right conditions, there doesn't seem to be any change in the object's coloring despite small variations in your qualia.[10] But even if I'm wrong, all that's ultimately important is that there is *a* disambiguation of 'looks exactly the same color' according to which changes in qualia do not necessarily change how things look, because we can just stipulate that's the sense relevant to our puzzle. And it seems obvious to me that there is such a disambiguation. Otherwise there'd be no such thing as color constancy; because our qualia are always changing, the coloring of objects would always look unstable.

7. The Objection from Visual Noise

Here's the problem with the triangulation view: as long as two objects are sufficiently similar, and as long as there is a time limit, we can't know by a reliable, introspective method whether those objects look exactly the same color. The triangulation view therefore forces us to give up the intuition about accessibility.

At first, it might seem like there's an easy way to establish this conclusion. Comparisons with additional objects take time. Moreover, there's no guarantee that all the relevant comparisons, considered together, won't take an extraordinary amount of time. One might think that's enough to establish that the triangulation view forces us to give up the intuition about accessibility.

However, it isn't that easy. We need to show that the comparisons might take an *extraordinary* amount of time rather than just a few seconds. Perhaps there's a procedure for quickly gathering likely candidates that allows us to stay within any time limit. We also need to show why the comparisons will *usually* take too long. Just as the possibility of failure doesn't imply that a process is unreliable, the possibility of delay doesn't imply that a process isn't usually quick, and that's all the intuition requires. Finally, we'd like to establish that comparisons will take too long *regardless of the circumstances*. Otherwise the triangulationist might still claim to preserve the spirit of the intuition about accessibility by showing that, in the right circumstances, we can know how things look.

I will argue that if the original objects are close enough in color, then even in the *ideal* circumstance—when we're immediately handed a chip that's discriminable from only one of the original objects—it would still *usually* take us an *extraordinary* amount of time to reach that conclusion. My objection will center on the fact that, due to visual noise, there is a limitation on our ability to know whether two things are discriminable within a given time limit.

[10] This is consistent with representationalism. Even if changes in individual qualia don't necessitate a change in what colors your experience represents, changes in the *frequency* of different qualia during a sufficiently long *interval* (e.g. one second) might still necessitate a change in what colors your experience represents.

As a helpful simplification, let's start by considering a task involving coins. Suppose that Tamar is asked to sort a pile of coins into two groups: coins that are biased tails and coins that aren't. Because some coins are more biased than others, it will be helpful to assign each coin a number between 0 and 1. In particular, let's assign 0 to coins that always land tails, 1 to coins that always land heads, and intermediate numbers to coins with intermediate biases. For example, let's assign $1/2$ to coins that land tails and heads with equal frequency. I will argue that if there is a maximum number of times that Tamar can flip any given coin, then there will be coins that she does not reliably categorize in any particular way.

How should Tamar decide whether a coin is biased tails? If she's rational, she'll use the following kind of heuristic: after flipping it a number of times she'll (1) believe that it's biased tails if it is sufficiently unlikely that a coin that isn't biased tails would produce that result, (2) believe that it isn't biased tails if it is sufficiently unlikely that a coin that is biased tails would produce that result, or (3) remain undecided and flip the coin again.[11]

How many times does Tamar need to flip a coin in order to reach a correct judgment? There is no fixed number of flips. Just as a fair coin might land on heads for the first hundred flips, and just as an extremely unfair coin might land heads and tails with equal frequency for the first hundred flips, it is *possible* for any coin to produce any series of results.

The question therefore needs to be refined: How many times does Tamar need to flip a coin in order to *reliably* reach a correct judgment? The answer is an ugly mathematical formula. Fortunately, all that is important for our purposes is that as the

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[11] Here's one way to make this more precise. After counting t tails after n flips:

(1) Believe that the coin is biased tails if:
For every coin that is not biased tails (that is: bias $\geq \frac{1}{2}$), the probability of getting t tails after n flips is less than .2.
(2) Believe that the coin isn't biased tails if:
For every coin that is biased tails (that is: bias $< \frac{1}{2}$), the probability of getting t tails after n flips is less than .2.
(3) Otherwise, remain undecided and flip again.

As I said above, this is just supposed to be an example of the *type* of heuristic Tamar should use. She might want to adjust the threshold of .2 depending on her prior credences about how many coins have each bias, whether she places more value on a speedy decision or a correct decision, and whether she is more worried about false positives or false negatives. If she values quick decisions, she'll replace the value of .2 in (1) and (2) with a higher value, and if she values correct decisions then she'll replace the value of .2 in (1) and (2) with a lower value. If she cares more about false negatives, she'll replace .2 so that the value in (2) is higher, and if she cares more about false positives, she'll replace .2 so that the value in (1) is higher. And if, for example, she believes that most of the coins have biases between $\frac{1}{3}$ and $\frac{1}{2}$, she should replace the .2 in (1) with a smaller number. She might also want to use a more sophisticated statistical method, perhaps using standard deviations rather than constants to precisify when it is 'sufficiently unlikely' that a coin with a certain bias would produce a certain result. But none of these modifications make any difference to the argument.

bias of a coin approaches $1/2$ from either direction, there's an increase in the number of flips Tamar needs to take in order to be reliable at sorting it. For example, suppose that after *n* tosses of a coin, the coins with biases in the shaded region will be *reliably* categorized as biased heads or not:

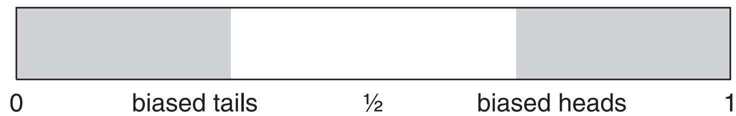

Figure 1

After $n+1$ tosses, more coins will be reliably sorted:

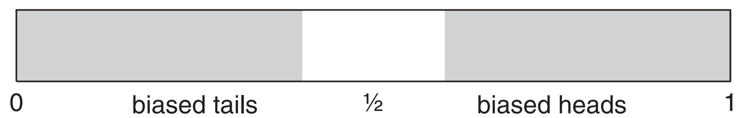

Figure 2

And after $n+2$, still more coins will be reliably sorted:

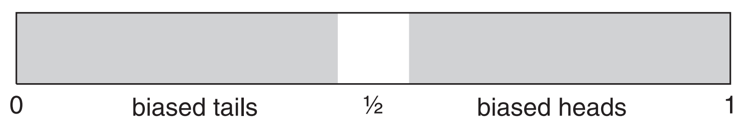

Figure 3

etc. The limit case is when a coin has a bias of exactly l_2. For such coins there is no finite number of flips after which Tamar will be reliable at sorting it. Of course, she might get lucky, because a coin with a bias of l_2 *might* produce a series of results that leads her to correctly and quickly categorize it as not biased tails. But after finitely many flips it is more likely that she will either make a false judgment or fail to make any judgment whatsoever.[12]

As a way of eliminating the gap, one might wonder why Tamar doesn't merely take her best guess when she reaches the maximum number of flips. I'm going to treat this as an objection and consider it later, after I've laid out the main argument.

[12] This is related to the strong law of large numbers. One difference is that we're interested in the number of flips after which you'd be *reliable* at sorting a given coin. Applied to coins, the strong law of large numbers says that if you flip a coin long enough, at some point you'll reach the right answer and flipping the coin more times won't change your mind. It doesn't imply that there is a number of flips such that if you flip the coin that many times you'll definitely reach the right answer. More importantly, it also doesn't imply that there is a number of flips such that if you flip the coin that many times you'll *reliably* reach the right answer.

I conclude that if there is a maximum number of times that Tamar can flip any given coin, then there will be a gap between (i) the coins she reliably believes are biased tails and (ii) the coins she reliably believes aren't biased tails.

This example teaches us a general lesson: When there's random variation in our measurements, the closer something is to a boundary, the more measurements we'll need to take in order to be reliable at judging that it is on one side of the boundary rather than the other. Therefore, if there is a limit on the number of measurements, there will be a gap between what we can know is on one side of the boundary through a reliable method and what we can know is on the other side of the boundary though a reliable method. I'm going to use this general lesson to identify a problem for the triangulation response.

As a first step, here's a chart that spells out the parallels between Tamar's task and Jonah's task:[13]

Tamar's task		Jonah's task
coins	→	objects
bias	→	color
biased heads	→	discriminable from given chip
series of flips	→	series of experiences
heads/tails	→	qualia

Figure 4

In sentences: Tamar is trying to decide whether a coin is biased heads by observing whether it lands heads or tails on successive flips. Jonah is trying to decide whether an object has a color that's discriminable from a given object by introspecting the qualia of his successive experiences of that object.

What is it for two colors to be discriminable? There is a range of views. While my argument won't presuppose any particular view, it will still be helpful to survey the options. On one extreme is the view that two colors are discriminable only if one can *know* that they are different. According to this view, to know that two colors are discriminable is to have a kind of higher-order knowledge: it is to know that one can know they are different colors. Williamson (1990) adopts this view. On

[13] This chart brings out a difference between Tamar's task and Jonah's task that might be distracting. In Jonah's task we can separate two elements: the colors of the chips and the visual noise that randomizes his experiences. In Tamar's task the corresponding elements are combined: the biases of the coins are both what she's trying to discover and what's responsible for randomizing her measurements. But this difference is incidental. From a mathematical point of view, it doesn't matter whether these two elements are combined or seperated. More specifically: There is a one-to-one correspondence between the relevant colors and the probability distributions giving the likelihood that those colors will produce experiences with different color qualia. Therefore, from a mathematical point of view, it doesn't matter if Jonah is interested in colors or probability distributions—the result is the same. We can therefore use Tamar's task as a guide to Jonah's task.

the other extreme is the view that whether two things are discriminable is simply a function of *reliability*. For example, according to one variant of this view, two colors are discriminable just in case one would say that they are different in more than fifty per cent of cases in a specified context, such as a standard forced-choice task. According to this view, to know that two colors are discriminable is to know something about one's own reliability. Most vision scientists adopt this view.

A weakness in many discussions of similar puzzles is that they assume a that particular view about discrimination is correct, thereby limiting the scope of their conclusions.[14] Luckily, we don't need to worry about which view is correct. For any given color there will be a boundary between the colors indiscriminable from it and the colors discriminable from it.[15] Therefore, when trying to decide whether a second color is discriminable from it, Jonah is trying to determine whether the second color is on one side of the boundary or the other.

We can now apply the general lesson to Jonah. If there is visual noise and if there is a maximum number of measurements by his visual system due to a time limit, then there will be a gap between (i) the colors Jonah reliably believes are discriminable from it and (ii) the colors Jonah reliably believes are indiscriminable from it. Recall that it's part of the intuition that motivates the triangulation view that we're reliable in the following sense:

> We know that two objects look exactly the same color in a context by a *reliable method* only if we're sufficiently likely to believe those objects look exactly the same color in that context using that method.

Given this condition, there will be a gap between (i) the colors Jonah can know by a reliable, introspective method are discriminable from it and (ii) the colors Jonah can know by a reliable, introspective method are indiscriminable from it. In a moment I will use this gap to identify a problem with the triangulation response.

It is worth emphaszing that this gap is due to two factors: visual noise and a time limit. Let's start with visual noise. Just as the biases of Tamar's coins introduce an element of uncertainty into her decisions, making it increasingly difficult to sort coins near the border, visual noise introduces an element of uncertainty into Jonah's decisions, making it increasingly difficult to sort colors near the discriminable-indiscriminable border. When there's random variation, the closer something is to a border, the more likely it is to behave like something on the other

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[14] For example, see Hardin, 1988, p. 218, and Hellie, 2005, p. 483.

[15] I'm suppressing a complication. There are views according to which, for a given color, there is a gap between the colors that are discriminable from it and the colors that are indiscriminable from it. In that case, because knowledge is factive, it would be easy to establish that there is a gap between the colors *that we can know* are discriminable from it and the colors *that we can know* are indiscriminable from it. And, as I'll argue later, that's enough to undermine the intuition about accessibility. I'm therefore focusing on the more challenging possibility that there is no gap between what's discriminable and what's indiscriminable.

side of the border, whether it is a coin landing on tails six times out of ten despite being slightly biased towards heads, or a color that causes a series of experiences that's often caused by colors on the other side of the discriminable-indiscriminable border. Random variation thereby creates uncertainty, increasing the probability that we will make an incorrect decision or fail to make any decision, especially near a border. By imposing a time limit we then create a gap between what we can know is on one side of the border and what we can know is on the other side of the border. Why impose a time limit? Recall that one feature of the intuiton about accessibility is that it doesn't take us an *extraordinary* amount of time to know whether two things look exactly the same color. Therefore, anyone trying to preserve this intuition, including the triangulation theorist, must acknowledge a time limit. For our purposes the value of that limit won't matter; it could be thirty seconds, five minutes, two hours, one year, or longer. It is enough that there is a limit.

We just used a general fact about measurements to establish that for any color there must be a gap between (i) the colors Jonah can know by a reliable, introspective method are discriminable from it and (ii) the colors Jonah can know by a reliable, introspective method are indiscriminable from it. This gap creates a problem for the triangulation response. Before I develop it, a terminological simplification: My argument is about whether we can know *by a reliable, introspective method* that two objects look exactly the same color *in less than an extraordinary amount of time*. To streamline the discussion I'll usually leave the italicized phrases implicit.

To make things easier, let's just focus on shades of gray, ordered from light to dark, setting aside other dimensions of variation. Let's suppose that, with respect to the colors along the spectrum below, the first chip's color is indiscriminable from shades between gray′ and gray″ and discriminable from the rest.

Figure 5

Let's next suppose that the second chip's color is indiscriminable from shades between gray* and gray** and discriminable from the rest:

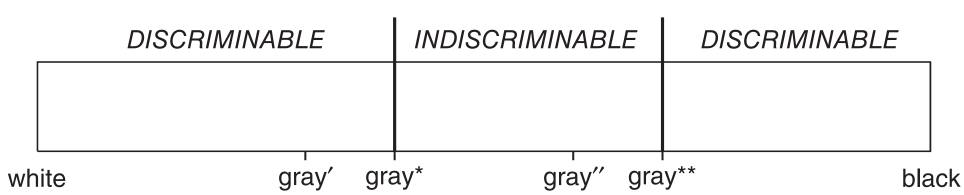

Figure 6

According to these Figures, there are colors that are discriminable from the second chip's color but not the first chip's color. In particular, shades that fall between gray′ and gray⋆ are discriminable from the second chip's color but indiscriminable from the first chip's color.

Here's the problem: As we already established, due to visual noise there will be a gap between the colors that Jonah can *know* are discriminable from a given chip's color and the colors that Jonah can *know* are indiscriminable from that chip's color. Let's use shading to indicate the colors that Jonah can know are discriminable or indiscriminable from the relevant chip. The first chip:

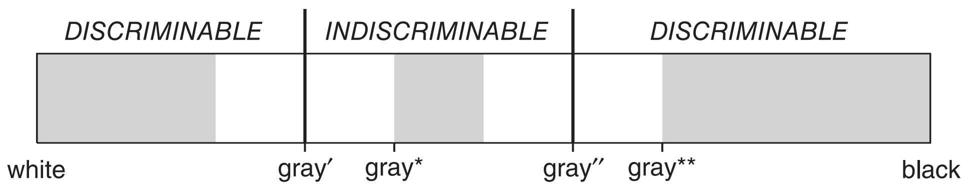

Figure 7

The second chip:

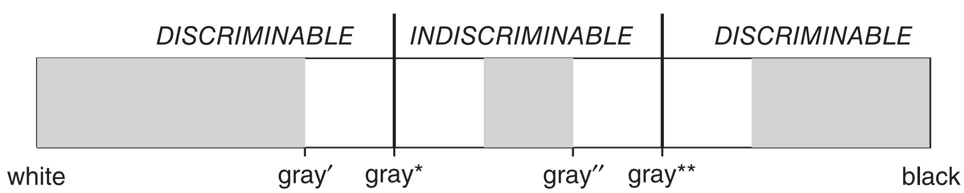

Figure 8

We can now state the core of the objection. According to these Figures, there are no colors that Jonah can *know* are discriminable from the color of the second chip and *know* are not discriminable from the color of the first chip. In particular, the only colors that are discriminable from the second chip's color but indiscriminable from the first chip's color are between gray′ and gray⋆, but if a shade falls between gray′ and gray⋆ then Jonah can neither know it is discriminable from the second chip's color nor know that it is indiscriminable from the first chip's color. For parallel reasons, there are no colors that Jonah can *know* are discriminable from the color of the first chip and *know* are indiscriminable from the color of the second chip. Thus: Jonah cannot know that the first and second chips *do not* look exactly the same color.[16]

[16] Can he know that they *do* look exactly the same color? Because knowledge is factive, that would imply that they look exactly the same color. Given that there are colors discriminable from only one of the chips' colors, that would be an odd thing for a triangulation theorist to say. But,

As I hope this example made clear, if we endorse the triangulation response, then for every object there will be another object with a similar but different color such that we can't know whether they look exactly the same color. How similar does the color of the other object have to be? It depends on the time limit (thirty seconds, five minutes, two hours, one year), the threshold for discrimination (fifty per cent, seventy perc ent, ninety-nine perc ent), and the amount of noise in one's visual system, among other factors. Regardless, it is enough that there will always be such an object, because that's enough to generate a Jonah-like series in which each object is close enough in color to its neighbor that we can't know that they look different even if we take into account comparisons with additional objects. The triangulation response therefore forces us to give up the intuition that all of us, including Jonah, can know whether neighboring objects look exactly the same color.

8. Discussion

There's a lot to say about this objection.

First, we just established that in the *ideal* circumstance, when we're immediately handed a chip that's discriminable from only one of the original chips, it would still *usually* take us an *extraordinary* amount of time to know that they do not look exactly the same color (regardless of where the triangulation theorist wants to draw the line between ordinary and extraordinary amounts of time). Our objection is therefore more satisfying than merely pointing out that it might take a while to find a chip that's discriminable from only one of the chips under consideration.

Second, it is worth reminding ourselves why the existence of noise in the visual system is responsible for the problem. Specifically, if it weren't for noise, there might be a line such that within the time limit we could know that colors on one side of the line are discriminable from a given color and we could know that colors on the other side of the line are not discriminable from the given color. In that case, the triangulation response would work, and it would give us a way of responding to the puzzle that preserves our intuition about accessibility. In the context of our example, we could know that the colors between gray′ and gray* are discriminable from only one of the chips' colors. But with noise, that's something we can't know.

Third, it is also worth reminding ourselves why the problem doesn't depend on one's view of discrimination. It is enough that there is a boundary between what's discriminable and what's indiscriminable. For the same reason, the problem remains even if the puzzle is reformulated using another notion, like matching, agreement, or distinguishability, because there would still be a boundary, and therefore there would still be a gap between what we can know is on one side of the boundary and what we can know is on the other side of the boundary.

regardless, if they look exactly the same color despite having different colors, then by iteration and TRANSITIVITY we could establish that all objects look exactly the same color, which is absurd. We can therefore set aside this possibility.

Fourth, the problem doesn't depend on any assumptions about how much, if any, of the relevant decision-making takes place at a 'personal level' as opposed to a 'sub-personal level'. It doesn't matter if one is consciously and deliberately recording the qualia of each experience and then calculating the probability that it is discriminable from a given color. It also doesn't matter if that calculation is unconsciously and automatically performed by a sub-personal module. The problem extends to any decision-maker regardless of whether it is a person, sub-personal module, or machine.

Fifth, it is worth acknowledging some questionable assumptions that we implicitly granted the triangulation theorist in order to distill the problem to its essence. Listing these assumptions will help clarify what assumptions are responsible for the problem (specifically: visual noise, time limit) and why dropping the other implicit assumptions would make things even worse for the triangulation theorist. We assumed that the qualia of Jonah's experiences are transparent in that he can always know exactly which qualia his experience currently instantiates. Dropping this assumption would be like asking Tamar to sort coins even if she's never sure whether a coin just landed heads or tails; it would further impair performance. We also assumed that Jonah knows that colors lighter than gray′ are discriminable from the color of the first chip and colors darker than gray′ are indiscriminable from the color of the first chip. But that's something Jonah could know only if he already knew the color of the first chip. Dropping this assumption would introduce more uncertainty into Jonah's decision making, worsening his performance. It would be like asking Tamar to pick out coins with biases less than the bias of a given coin without telling her the bias of that coin. Further, we focused on one dimension of color variation even though, as noted earlier, there are several. Because there will be noise along every dimension, that introduces more unknown variables, further complicating Jonah's task, making it even harder to know whether two colors are discriminable. Finally, we ignored the fact that 'discriminable' and 'indiscriminable' are vague (see Varzi, 1995, for discussion), a fact that would further impair Jonah's decisions.

Let's now consider and dismiss two likely responses by triangulation theorists. Even given our conclusion, it's possible there are objects whose color is similar enough to the first chip's color that by direct comparison we can't know they look differently colored, but whose color is dissimilar enough to the first chip's color that we can know they look different colors by comparing them to additional objects. Perhaps we can divide objects into three categories: (i) those we can know look different from the first chip *by direct comparison*, (ii) those we can know look different from the first chip only *by comparison to additional objects*, and (iii) those whose relationships to the first chip are unknowable. Supposing that the first chip is red_{264}, we might divide colors along one dimension through color space:

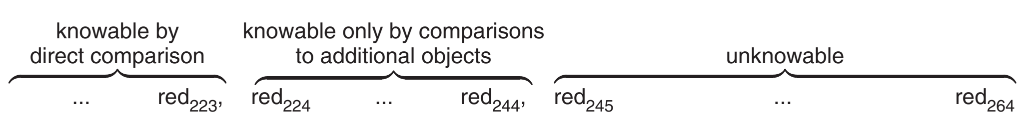

Figure 9

This possibility shouldn't give the triangulation theorist much comfort. Due to the third category, represented by the far-right bracket, there are sill Jonah-type series where comparisons to additional objects won't help us know that neighbors look different. The objects in that series just need to have sufficiently similar colors. When confronted with such a series the triangulation theorist must give up our intuition about accessibility, the very intuition her response was designed to preserve. If we take that intuition seriously, then we should try to find a response that handles *all* series without abandoning this intuition. I explore several of these responses in (Morrison, 2015).

A triangulation theorist might also wonder why Jonah, or a sub-personal module in his visual system, doesn't just guess at the time limit. One might hope that would eliminate the gap. Two problems, both decisive: First, the intuition is that we're *reliable*, and that's not just the intuition that we're accurate in more than fifty per cent of cases. Intuitively, we're accurate more often than that. Therefore, even if Jonah takes his best guess when he reaches the time limit, there will be chips such that he can't know whether they are discriminable. For example, if reliability requires at least seventy-five per cent accuracy, then there will be lots of chips such that, even with guessing, Jonah won't be reliable. At the limit is a chip whose color is on the border between what's discriminable and what's indiscriminable, because, even with guessing, Jonah will judge with equal frequency that it is discriminable and that it is indiscriminable. Second, the intuition is that we can *know* when things look exactly the same color, and you can't know something as the result of a guess.

9. Conclusion

Our objection improves on existing objections. We already compared it to Dummett's objection. Let's end by comparing it to three other objections.

Fara (2001, p. 915), Varzi (1995, p. 51), and Wright (1975, p. 355) all object that the triangulation view implies that 'look exactly the same color' isn't an observational predicate. According to the standard definition, an *n*-place predicate is observational if and only if we can know whether *n* objects satisfy it just by observing those *n* objects. The triangulationist claims that we can know whether two objects satisfy the predicate 'look exactly the same color' only after observing their relations to additional objects. But, the objection concludes, it is obvious that 'look exactly the same color' is an observational predicate.

This objection is unlikely to change anyone's mind. The predicate 'look exactly the same color' would still be observational *in some sense* if you could know that two objects satisfy it by observing them together with a third object. If the triangulation view solved the puzzle, that would seem like an insignificant adjustment. Moreover, some argue that to know whether objects satisfy a predicate, we always need to rely on a background theory; observation alone isn't enough (famously, Quine, 1951). They sometimes claim that predicates are observational to various degrees, always less than one. The objection to the triangulation view would then lose its bite, because, at worst, the triangulation view would imply that 'look exactly the same color' is less observational than we originally thought. Perhaps these authors can convince us that this has unacceptable epistemological or semantic consequences. But I'm doubtful. Moreover, this objection is at odds with the way people normally determine how things look. When deciding whether an article of clothing looks navy, one might compare it to something that looks black. When deciding whether a wall looks white-and-illuminated-by-red-light, rather than red-and-illuminated-by-colorless-light, one might compare it to nearby objects to gauge whether the illuminant is red or colorless. Given that comparisons with other objects often help us determine how things look, we should at least be open to the possibility that they sometimes play a role in determining whether two things look exactly the same color.

The second objection is that the triangulation view implies that we can never *know* that two objects look exactly the same color, because we can't rule out the possibility that, if we looked around more, we'd find an object whose color is discriminable from the color of only one of the original objects. This is a familiar kind of objection; similar doubts can be raised for any apparent case of inductive knowledge. The triangulation theorist shouldn't worry about this objection, for the same reason she shouldn't worry about external-world skepticism; if it turns out that there are no satisfying answers to this kind of objection, we'll have bigger worries than the failure of the triangulation response.

The third objection is that, given that we need to look around, the triangulation response does not allow us to know *by introspection* that two things look exactly the same color. But this objection is too quick. We can know by observation that Mount Washington is tall even if we need to drive there to see it. That's because our driving is just a causal precondition—our observation of Mount Washington is what ultimately justifies our belief. Likewise, according to some philosophers, we can know *a priori* that geometrical claims are false even if we first need to look at diagrams of counterexamples. That's because our looking at these counterexamples is just a causal precondition—our geometric understanding is what ultimately justifies our belief (Burge [2000, p. 17] attributes this view to Frege). A proponent of the triangulation response might similarly insist that we can still know by introspection that two objects look exactly the same color as long as the search is just a causal precondition—our introspection of the resulting experiences is what ultimately justifies our belief.

Just to be clear: I'm not claiming that these objections are without merit. I'm just claiming that our objection is an improvement. As I said, it's important to reject the wrong view for the right reasons.

Department of Philosophy
Barnard College, Columbia University

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